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Least Squares Solution for System Identification with Non-uniform Data under a Coprime Collaborative Sensing Scheme

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***Abstract*— This paper presents a least squares formulation and a closed-form solution for identifying dynamical systems using non-uniform data obtained under a coprime collaborative sensing scheme. Specifically, the method combines measurements from two slow output sensors with different sampling rates to estimate the system's dynamics. We provide the theoretical foundation for developing advanced least-squares-based system identification algorithms for cases where the input-output data are sampled at different rates. Demonstrative examples are provided to validate the proposed method, and to show the identification beyond the Nyquist frequency.**

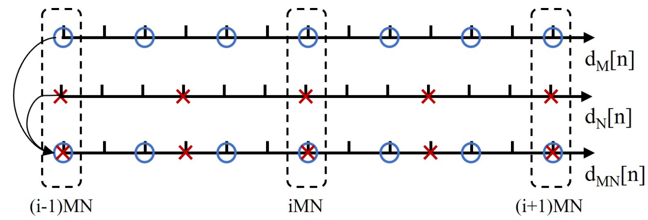

Fig. 1. Illustration of the irregular and sparse measurements collected by two sensors at different sampling rates (e.g., $M = 2$, $N = 3$). Parameter estimation is only valid at instants of an integer multiple of MN.

I. INTRODUCTION

System identification from sampled data is an underpinning for understanding and controlling dynamic systems. Standard system identification algorithms require uniformly sampled data usually collected at a consistent sampling rate between the input and the output [1]. Non-uniform data, in contrast, is thought to possess higher temporal resolutions, which can help to alleviate or even remove the adverse effect of aliasing when the average sampling rate falls below twice the desired Nyquist frequency of the target dynamics [2]. However, non-uniform measurements pose a challenge for most system identification algorithms, and it remains not well understood how to perform system identification to fully realize the benefits of non-uniform data. In this paper, we propose a novel coprime collaborative sampling scheme that systematically generates non-uniform measurements, together with a polynomial-transformation-based model reparameterization technique that ultimately enables model identification beyond the conventional Nyquist limit. The main objective is to optimize the efficacy of multiple sensors for applications where the sensor speed is slow compared to the monitored process, which is commonly referred to as mixed-rate systems. By effectively managing sensors, the proposed approach aims to address the limitation of existing methods and enable previously infeasible applications in non-contact sensing systems such as laser processing and vision-based inspection.

Mixed-rate (MR) system identification has been discussed over decades due to its various applications such as battery modeling, quadcopter dynamics modeling, and fault detection [3]–[6]. From a signal processing perspective, one may still be able to use single-rate system identification algorithms to identify MR system models by matching the sampling rates through downsampling or upsampling the collected measurements, which are widely used in the industry. However, downsampling can lead to information loss and potentially cause aliasing [7]. While upsampling can introduce additional noise or artifacts into the signal, potentially distorting or obscuring the true system dynamics as the added samples are generated solely based on the chosen interpolation method without direct knowledge of the underlying system dynamics [8]. Even though they enable the use of single-rate identification algorithms in MR systems, neither guarantees accurate identification of the full system dynamics. Instead of matching the sampling rate, the polynomial transformation technique [9] offers an alternative to directly identifying the system model, which also provides the theoretical basis for the realization of the utilization of multiple sensors in MR system identification. Previously, Ding *et al.* [10] investigated collaborative sensing strategies in reconstructing the continuous-time systems from a discrete-time system subjected to irregular sampling rates, which in some senses proved the feasibility of identifying the system model with non-uniformly sampled data.

The proposed coprime collaborative sensing in this article leverages two slow sensors of different sampling rates that are systematically timed and processed for identifying system dynamics at orders of speed beyond the individual sensors (Fig. 1). Such a collaborating sampling maintains the individual sensor's uniform sampling routine while adding information beyond the individual sensor's sensing capability, leading to an overall non-uniform sampled data collection at a greater temporal resolution.

Methodology wise, part of the system identification algorithm is motivated by the polynomial transformation method in MR system analysis. Two general approaches exist when dealing with MR systems: the lifting technique [11], and the polynomial transformation method [9]. The lifting technique is a state-space approach that translates the MR system into a

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single-rate system [12], while the polynomial transformation method enables the direct identification of the MR system model by mathematically manipulating the characteristic polynomial in the transfer-function domain. Intensive efforts have been put into the first approach [13]–[15], while the latter remains intriguing due to advantages in real-time implementations. Based on the proposed coprime collaborative sensing scheme, this paper introduces the design and application of transformation polynomials to reparameterize a candidate system model. Through reparameterization, the general form of the MR system is obtained. Next, a least squares optimization is formulated to obtain the intermediate parameters of the reparameterized transfer function. Finally, parameter post-processing is implemented to recover the fast system model.

The remainder of this paper is organized as follows: Section II introduces technical preliminaries. In Section III, we present the least squares formulation in matrix form and its closed-form solution under the proposed coprime collaborative sensing scheme. A first-order and a second-order system identification example are provided in Section IV, and conclusions are made in Section V.

II. PRELIMINARIES

In this section, we first review the transfer function approach for general system identification based on the ARMA model. Then, we show the polynomial transformation method for a class of system identification problems when the input and the output are sampled at different rates.

A. Transfer function approach for single-rate system identification

Consider a general stable ARMA model [16]

$$y(k) = G(q^{-1})u(k) = \frac{q^{-d}B(q^{-1})}{A(q^{-1})}u(k) \tag{1}$$

where d is an integer number representing the pure time delay of the system (excluding a one-step intrinsic delay of the discrete-time system), q^{-1} is the one-step delay operator, and

$$\begin{aligned} A(q^{-1}) &= 1 + a_1 q^{-1} + \cdots + a_{n_a} q^{-n_a} \\ B(q^{-1}) &= b_1 q^{-1} + b_2 q^{-2} + \cdots + b_{n_b} q^{-n_b} \end{aligned}$$

are monic polynomials of q^{-1}. We assume that the model is irreducible (i.e., $q^{n_a}A(q^{-1})$ and $q^{n_b}B(q^{-1})$ are coprime). Introduce

$$\begin{aligned} A^*(q^{-1}) &= a_1 + a_2 q^{-1} \cdots + a_{n_a} q^{-n_a+1} \\ B^*(q^{-1}) &= b_1 + b_2 q^{-1} + \cdots + b_{n_b} q^{-n_b+1} \end{aligned}$$

where $A^*(q^{-1}) = q[A(q^{-1}) - 1]$ and $B^*(q^{-1}) = qB(q^{-1})$. Then, Eq. (1) can be rewritten as

$$y(k) = \frac{q^{-d-1}B^*(q^{-1})}{1 + q^{-1}A^*(q^{-1})}u(k) \tag{2}$$

namely

$$(1 + q^{-1}A^*(q^{-1}))y(k) = q^{-d-1}B^*(q^{-1})u(k)$$

or

$$\begin{aligned} y(k) =& - a_1 y(k-1) - a_2 y(k-2) - \cdots - a_{n_a} y(k-n_a) \\ &+ b_1 u(k-d-1) + \cdots + b_{n_b} u(k-d-n_b) \end{aligned}$$

In vector form

$$y(k) = \theta^T \phi(k) \tag{3}$$

where

$$\begin{aligned} \phi(k) = [&-y(k-1), -y(k-2), \ldots, -y(k-n_a), \\ &u(k-d-1), u(k-d-2), \ldots, u(k-d-n_b)]^T \\ \theta = [&a_1, a_2, \ldots, a_{n_a}, b_1, b_2, \ldots, b_{n_b}]^T \end{aligned}$$

B. Polynomial transformation for dual-rate system identification

Consider a dual-rate system that has its output $y(kLT)$ sampled L times slower than its input $u(kT)$ where T is the fundamental sampling period. Let $T = 1$ for simplicity in the rest of the content. We first recognize the factorization

$$1 - x^L = (1-x)(1 + x + x^2 + \cdots + x^{L-1}) \tag{4}$$

and the characteristic equation $A(q^{-1})$ in its product form

$$A(q^{-1}) \triangleq \prod_{i=1}^{n_a}(1 - (\lambda_i q)^{-1}) \tag{5}$$

where λ_i's are the reciprocals of the poles of the system model that needs to be identified, and n_a is the order of the characteristic equation (i.e. the number of poles) of the system. Define the transformation polynomial:

$$\begin{aligned} F_L(q^{-1}) &= \prod_{i=1}^{n_a}(1 + (\lambda_i q)^{-1} + (\lambda_i q)^{-2} + \cdots + (\lambda_i q)^{-L+1}) \\ &= 1 + f_1 q^{-1} + \cdots + f_{n_a L - n_a} q^{-n_a L + n_a} \end{aligned}$$

Referring to Eq. (4), Eq. (5) satisfies

$$A(q^{-1})F_L(q^{-1}) = \prod_{i=1}^{n_a}(1 - (\lambda_i q)^{-L})$$

where the order of the shift operator q becomes an integer multiple of L. By doing so, we are able to manipulate the order of the shift operator and then change the required data for the identification process. By applying the same transformation polynomial to $B(q^{-1})$, Eq. (1) can be rewritten as:

$$y(k) = \frac{q^{-d}B(q^{-1})F_L(q^{-1})}{A(q^{-1})F_L(q^{-1})}u(k) \triangleq \frac{q^{-d}B_L(q^{-1})}{A_L(q^{-L})}u(k)$$

where

$$\begin{aligned} A_L(q^{-L}) &= 1 + a_{L,1} q^{-L} + a_{L,2} q^{-2L} + \cdots + a_{L,n_a} q^{-n_a L} \\ B_L(q^{-1}) &= b_{L,1} q^{-1} + \cdots + b_{L,n_a(L-1)+n_b} q^{-n_a(L-1)-n_b} \end{aligned}$$

For notational convenience, let $\varkappa = n_a(L-1) + n_b$. The system model can be re-written in the form of Eq. (2)

$$y(k) = \frac{q^{-d-1}B_L^*(q^{-1})}{1 + q^{-L}A_L^*(q^{-1})}u(k)$$

where

$$A_L^*(q^{-1}) = a_{L,1} + a_{L,2}q^{-1} \cdots + a_{L,n_a}q^{-n_a L+1}$$
$$B_L^*(q^{-1}) = b_{L,1} + b_{L,2}q^{-1} + \cdots + b_{L,\varkappa}q^{-\varkappa+1}$$

Therefore the output in a predictor-like structure can be written as follows:

$$y(k) = \theta_L^T \phi_L(k)$$

where

$$\phi_L(k) = [-y(k-L), -y(k-2L), \ldots, -y(k-n_a L), u(k-d-1), u(k-d-2), \ldots, u(k-d-\varkappa)]^T$$
$$\theta_L = [a_{L,1}, a_{L,2}, \ldots, a_{L,n_a}, b_{L,1}, b_{L,2}, \ldots, b_{L,\varkappa}]^T$$

The model introduced in Eq. (3) cannot be identified when the measurable data are $\{u(k), y(kL), k = 0, 1, 2, ...\}$. In contrast to a single-rate regressor form, this model reparameterization selectively downsamples the observation space and enables direct system identification using smaller amounts of uniformed sampled data. The original system parameters can be recovered from the reparameterized model by removing the common factor from the numerator and denominator (i.e. $F_L(q^{-1})$). Therefore, the fast model is identified from slow measurements.

III. Model Reparameterization for System Identification Under Collaborative Sensing

In order to use non-uniform output data to identify the system dynamics, for the case with two sensors, we design the transformation polynomials as follows

$$F_M(q^{-1}) = \prod_{i=1}^{n_a}(1 + (\lambda_i q)^{-1} + \cdots + (\lambda_i q)^{-M-1})$$
$$F_N(q^{-1}) = \prod_{i=1}^{n_a}(1 + (\lambda_i q)^{-1} + \cdots + (\lambda_i q)^{-N-1})$$

where M and N are coprime integers representing the ratios between the input sampling rate and the sampling rates of the fast and the slow sensors, respectively. Without loss of generality, we assume $N > M$. If not, we simply swap and rename the two sensors. M and N being coprime offers a greater temporal resolution compared to non-coprime setups where more measurements are overlapped. In Fig. 2 (c & d), we see the non-coprime setup results in measurements with a periodical pattern, while the coprime setup captures much more information about the underlying dynamics of the system.

Next, we combine two transformation polynomials and apply an additional normalization (division by a factor of 2), which yields

$$F_{MN}(q^{-1}) = \frac{1}{2}[F_M(q^{-1}) + F_N(q^{-1})] \tag{6}$$

Obviously, the formulation can be easily extended to multiple sensors as follows

$$\hat{F}(q^{-1}) = \frac{1}{n}[\sum_{i=1}^{n} F_i(q^{-1})] \tag{7}$$

Multiplying the combined transformation polynomial Eq. (6) to both the numerator and the denominator of the original transfer function Eq. (1) yields

$$y(k) = \frac{q^{-d}B(q^{-1})F_{MN}(q^{-1})}{A(q^{-1})F_{MN}(q^{-1})}u(k) \triangleq \frac{q^{-d}B_{MN}(q^{-1})}{A_{MN}(q^{-M}, q^{-N})}u(k)$$

In more details,

$$y(k) = \frac{q^{-d}B_{MN}(q^{-1})}{\frac{1}{2}[A(q^{-1})F_M(q^{-1}) + A(q^{-1})F_N(q^{-1})]}u(k)$$

where

$$A(q^{-1})F_M(q^{-1}) = \prod_{i=1}^{n_a}(1 - (\lambda_i q)^{-M}) = 1 + a_{M,1}q^{-M} + \cdots + a_{M,n_a}q^{-n_a M}$$
$$A(q^{-1})F_N(q^{-1}) = \prod_{i=1}^{n_a}(1 - (\lambda_i q)^{-N}) = 1 + a_{N,1}q^{-N} + \cdots + a_{N,n_a}q^{-n_a N}$$

After normalization, the characteristic equation of the reparameterized transfer function becomes

$$A(q^{-1})F_{MN}(q^{-1}) = 1 + \frac{a_{M,1}}{2}q^{-M} + \cdots + \frac{a_{M,n_a}}{2}q^{-n_a M} + \frac{a_{N,1}}{2}q^{-N} + \cdots + \frac{a_{N,n_a}}{2}q^{-n_a N}$$

Then, the reparameterized system model under the collaborative sensing scheme can be written as

$$y(k) = \frac{q^{-d}B_{MN}(q^{-1})}{1 + q^{-M}\frac{A_M(q^{-M})}{2} + q^{-N}\frac{A_N(q^{-M})}{2}}u(k)$$

where

$$B_{MN}(q^{-1}) = b_{MN,1}q^{-1} + b_{MN,2}q^{-2} + \cdots + b_{MN,\varkappa}q^{-\varkappa}$$
$$A_M(q^{-M}) = a_{M,1} + a_{M,2}q^{-M} + \cdots + a_{M,n_a}q^{-n_a M+M}$$
$$A_N(q^{-N}) = a_{N,1} + a_{N,2}q^{-N} + \cdots + a_{N,n_a}q^{-n_a N+N}$$

Noting that $\varkappa = n_a(N-1) + n_b$. The above model provides

$$y(k) = -\frac{1}{2}A_M(q^{-M})y(k-M) - \frac{1}{2}A_N(q^{-N})y(k-N) + B_{MN}(q^{-1})u(k)$$

or in a vector form

$$y(k) = \theta_{MN}^T \phi_{MN}(k) \tag{8}$$

where

$$\phi_{MN}(k) = \begin{bmatrix} -\frac{1}{2}y(k-M) \\ -\frac{1}{2}y(k-2M) \\ \vdots \\ -\frac{1}{2}y(k-n_aM) \\ \hline -\frac{1}{2}y(k-N) \\ -\frac{1}{2}y(k-2N) \\ \vdots \\ -\frac{1}{2}y(k-n_aN) \\ \hline u(k-d-1) \\ u(k-d-2) \\ \vdots \\ u(k-d-\varkappa) \end{bmatrix}, \quad \theta_{MN} = \begin{bmatrix} a_{M,1} \\ a_{M,2} \\ \vdots \\ a_{M,n_a} \\ \hline a_{N,1} \\ a_{N,2} \\ \vdots \\ a_{N,n_a} \\ \hline b_{MN,1} \\ b_{MN,2} \\ \vdots \\ b_{MN,\varkappa} \end{bmatrix}$$

From Eq. (8), we see that if we wanted to estimate output at time k, the output measurements at time $k-M$ and $k-N$ have to be available. This only works when k is an integer multiple of MN as shown in Fig. 1. With this in mind, suppose we have the estimation of the parameter vector $\hat{\theta}_{MN}$ at time $k = iMN$, a posterior output estimation will be

$$\hat{y}(iMN) = \phi^T(iMN)\hat{\theta}(iMN)$$

$$i = 0, 1, 2, \ldots$$

Consider the performance index

$$\begin{aligned} J_k &= \sum_{i=1}^{k} e(iMN)^2 \\ &= \sum_{i=1}^{k} [y(iMN) - \phi_{MN}^T(iMN)\hat{\theta}_{MN}(kMN)]^2 \end{aligned}$$

For simplicity, we use ϕ and θ to represent ϕ_{MN} and θ_{MN} in the remaining of this section. The summation in the performance index can be expanded and rewritten in a vector form as follows:

$$\begin{bmatrix} e(kMN) \\ e((k-1)MN) \\ \vdots \\ e(MN) \end{bmatrix} = \underbrace{\begin{bmatrix} y(kMN) \\ y((k-1)MN) \\ \vdots \\ y(MN) \end{bmatrix}}_{Y_k} - \underbrace{\begin{bmatrix} \phi(kMN) & \phi((k-1)MN) & \ldots & \phi(MN) \end{bmatrix}}_{\Phi_k^T} \hat{\theta}(kMN)$$

hence

$$\begin{aligned} J_k &= ||Y_k - \Phi_k\hat{\theta}(k)||^2 \\ &= (\hat{\theta}^T(k)\Phi_k^T\Phi_k\hat{\theta}(k) - 2Y_k^T\Phi_k^T\hat{\theta}(k) + Y_k^TY_k) \end{aligned}$$

The least squares solution targets to minimize the aforementioned performance index. Taking the partial derivative, we have

$$\frac{\partial J_k}{\partial \theta} = 2\Phi_k^T\Phi_k\hat{\theta}(k) - 2\Phi_kY_k$$

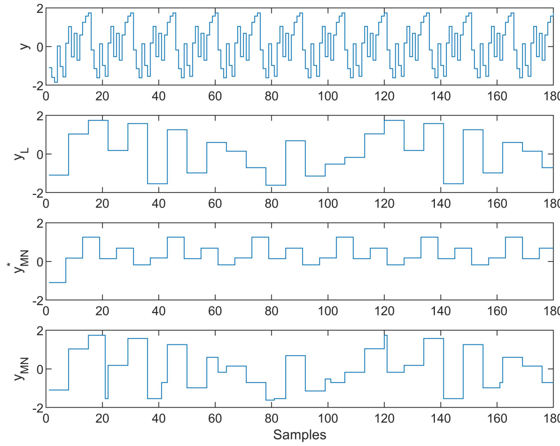

Fig. 2. From top to bottom: (a) samples needed for single-rate system identification algorithms. (b) samples needed when one slow sensor (e.g. $L = 7$) is used. (c) samples colllected when two non-coprime sensors (e.g.$M = 6$,$N = 18$) are used. (d) samples collected when two coprime sensors (e.g. $M = 7$,$N = 20$) are collaboratively taking measurements.

Letting $\frac{\partial J_k}{\partial \theta} = 0$ yields the unique optimal solution:

$$\hat{\theta}(k)_{opt} = (\Phi_k\Phi_k^T)^{-1}\Phi_kY_k \tag{9}$$

We have thus obtained the least squares solution for the case where two coprime sensors collaboratively sense the system response.

IV. Numerical Example

A. 1^{st} Order System:

For illustration purposes, consider the first-order system

$$\frac{B(q^{-1})}{A(q^{-1})} = \frac{1}{1-(\lambda q)^{-1}} \overset{e.g.}{=} \frac{1}{1-0.5q^{-1}}$$

where $\lambda = 2$ is the parameter to be identified.

Referring and comparing to the single-rate system model Eq. (1), the first order system's pure delay is $d = 0$; the order of the denominator polynomial $A(q^{-1})$ is $n_a = 1$; and the order of the numerator polynomial $B((q^{-1})$ is $n_b = 0$.

Two sensors are used for sampling the system output at different rates. One is M times slower than the input sampling rate and the other is N times slower. For this example, we assume $M = 7$ and $N = 20$.

Next, construct transformation polynomials as follows

$$\begin{aligned} F_M(q^{-1}) &= 1 + (\lambda q)^{-1} + \cdots + (\lambda q)^{-6} \\ F_N(q^{-1}) &= 1 + (\lambda q)^{-1} + \cdots + (\lambda q)^{-6} + \cdots + (\lambda q)^{-19} \end{aligned}$$

Implement the normalization as shown in Eq. (6)

$$\begin{aligned} F_{MN}(q^{-1}) =& \frac{1}{2}[F_M(q^{-1}) + F_N(q^{-1})] \\ =& 1 + (\lambda q)^{-1} + \cdots + (\lambda q)^{-6} \\ &+ \frac{1}{2}(\lambda q)^{-7} + \cdots + \frac{1}{2}(\lambda q)^{-19} \end{aligned}$$

and applying the normalized transformation polynomial to the system yields

$$y(k) = \frac{B(q^{-1})F_{MN}(q^{-1})}{A(q^{-1})F_{MN}(q^{-1})}u(k) = \frac{1+(\lambda q)^{-1}+\cdots+(\lambda q)^{-6}+\cdots+\frac{1}{2}(\lambda q)^{-19}}{1-\frac{1}{2}(\lambda q)^{-7}-\frac{1}{2}(\lambda q)^{-20}}u(k)$$

Re-writing in the predictor-like structure yields

$$\begin{aligned} y(k) = &-\frac{1}{2}\lambda^{-7}y(k-7)-\frac{1}{2}\lambda^{-20}y(k-20) \\ &+u(k)+\lambda^{-1}u(k-1)+\cdots+\lambda^{-6}u(k-6) \\ &+\frac{1}{2}\lambda^{-7}u(k-7)+\cdots+\frac{1}{2}\lambda^{-25}u(k-19) \end{aligned}$$

or equivalently in a vector form:

$$y(k) = \underbrace{\begin{bmatrix}\lambda^{-7}\\ \lambda^{-20}\\ \lambda^{-1}\\ \vdots \\ \lambda^{-6}\\ \lambda^{-7}\\ \vdots\\ \lambda^{-19}\end{bmatrix}^T}_{\theta^T}\underbrace{\begin{bmatrix}-\frac{1}{2}y(k-7)\\ -\frac{1}{2}y(k-20)\\ u(k-1)\\ \vdots\\ u(k-6)\\ \frac{1}{2}u(k-7)\\ \vdots\\ \frac{1}{2}u(k-19)\end{bmatrix}}_{\phi(k)} + u(k)$$

A pseudo-random binary sequence (PRBS) is used to excite the system. For evaluation purposes, the ground truth full system response (not available to the proposed system identification algorithm) is sampled at 1024 HZ, which is the same as the input sampling frequency. The least squares formation is as follows:

$$Y_k = \begin{bmatrix} y(k)\\ y(k-140)\\ \vdots\\ y(k-140n)\end{bmatrix} - \begin{bmatrix} u(k)\\ u(k-140)\\ \vdots\\ u(k-140n)\end{bmatrix}$$

$$\Phi_k = \begin{bmatrix} -\frac{1}{2}y(k-7) & \dots & -\frac{1}{2}y(k-7-140n)\\ -\frac{1}{2}y(k-20) & \dots & -\frac{1}{2}y(k-20-140n)\\ u(k-1) & \dots & u(k-1-140n)\\ \vdots & & \vdots\\ u(k-6) & \dots & u(k-6-140n)\\ \frac{1}{2}u(k-7) & \dots & \frac{1}{2}u(k-7-140n)\\ \vdots & & \vdots\\ \frac{1}{2}u(k-19) & \dots & \frac{1}{2}u(k-19-140n)\end{bmatrix}$$

and the parameter vector is in the following form

$$\hat{\theta}^T = \begin{bmatrix}\lambda^{-7} & \lambda^{-20} & \lambda^{-1} & \lambda^{-2} & \dots & \lambda^{-19}\end{bmatrix}$$

Therefore, the unique least squares solution can be found using Eq. (9), and the final λ estimation is then obtained by removing the greatest common factor between the denominator and numerator:

$$\hat{\lambda} = 2.0000$$

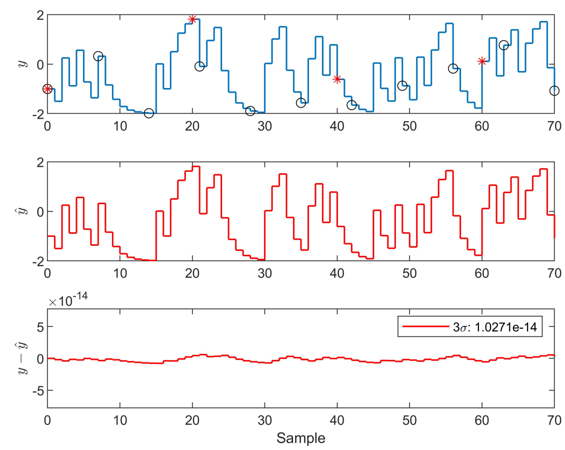

Fig. 3. Top: ground truth output with irregular and sparse sampling ($M = 7$, $N = 20$). Middle: the estimated output from the identified system model. Bottom: relative error.

In Fig. 3. we plot the actual samples we used for the system identification, the output estimation by using the identified model, and the output estimation error. The comparison between the real and identified system's frequency response is shown in Fig. 4. The Nyquist frequency of each output sensor (i.e. 19.6 Hz and 73.1 Hz) is also marked out in Fig. 4. The dynamics beyond the Nyquist frequency of each sensor is observed to have been successfully extracted.

B. 2^{nd} Order System:

The identification of a second-order system:

$$\frac{B(q^{-1})}{A(q^{-1})} = \frac{0.7047q^{-1}}{1-1.125q^{-1}+0.8129q^{-2}}$$

with pure delay $d = 0$; the order of the denominator polynomial $n_a = 2$; and the order of the numerator polynomial $n_b = 1$ is provided under the same sensor setup (i.e. $M = 7$, $N = 20$) and the same PRBS input signal sampled at 4.82 HZ. The system response is shown in Fig. 5. We observe that the mode around 0.7Hz is beyond each sensor's Nyquist frequency but well identified.

V. CONCLUSIONS

The paper presents a least squares formulation and closed-form solution for the coprime collaborative sensing scheme, and illustrates its effectiveness through two system identification examples. Future work will include developing a recursive-least-squares formulation under the coprime collaborative sensing and investigating criteria for sensor rate selection. The proposed principle can be easily extended to multiple sensor cases and slow-input-fast-output cases.

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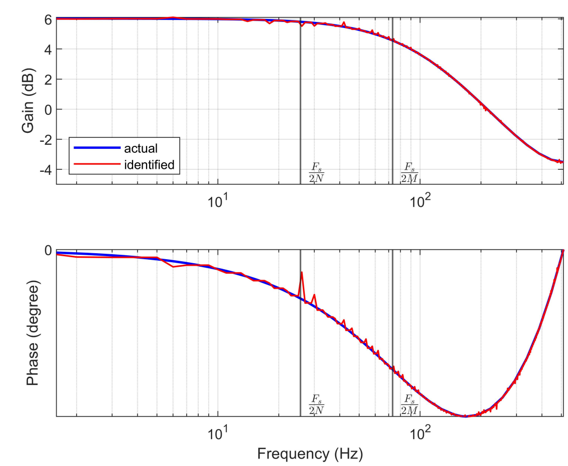

Fig. 4. The system response comparison between the real system and identified system

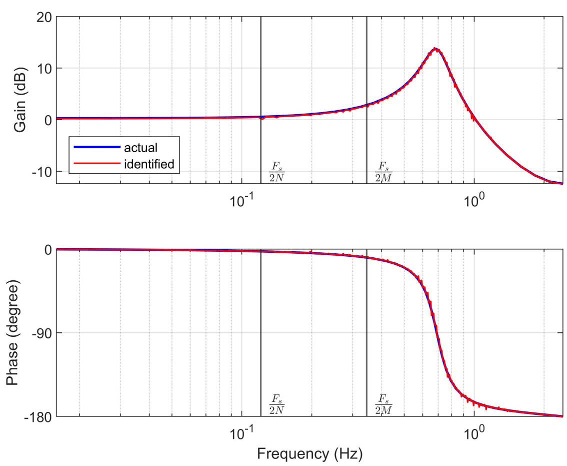

Fig. 5. The second-order system response between the real system and identified system

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